Introduction to Motion Planning

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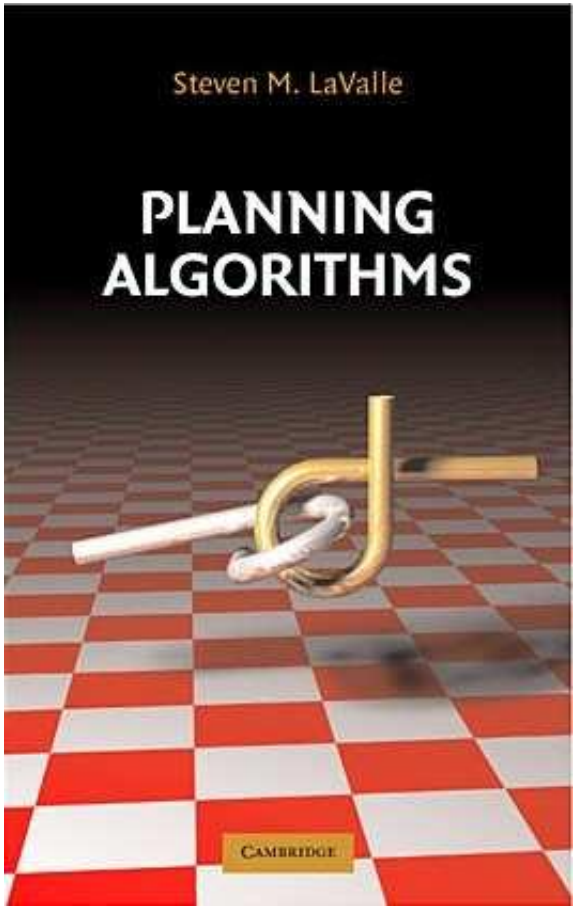
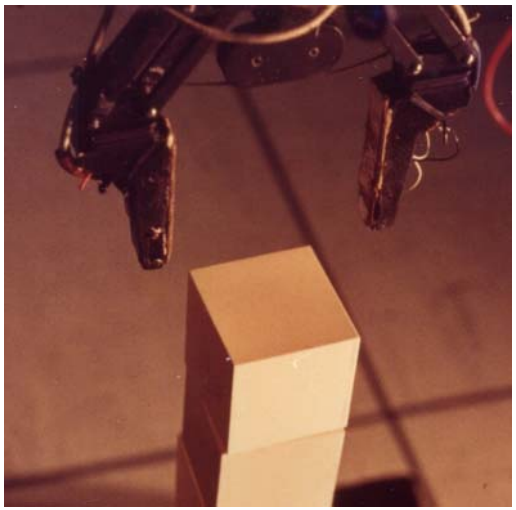
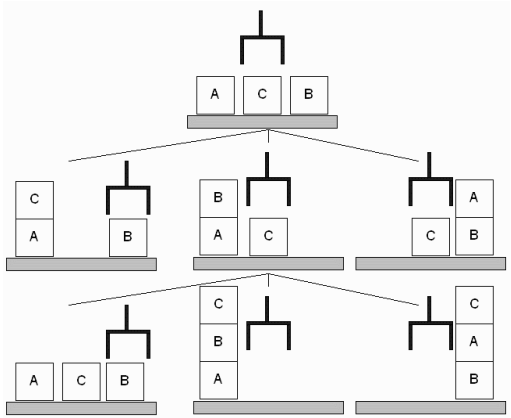
Overview

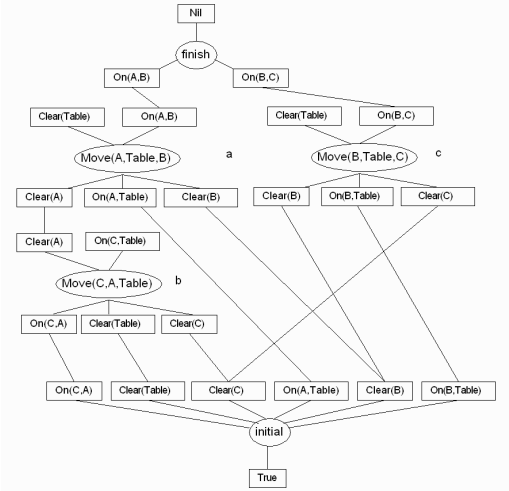
1. Motivation
   - Planning history
2. General concepts
3. Discrete planning
   - ...searching really
4. Modelling
   - configuration space
   - cell decompositions
5. Continuous planning
   - feedback motion planning
   - sampling based motion planning
   - extensions
6. Planning under differential constraints



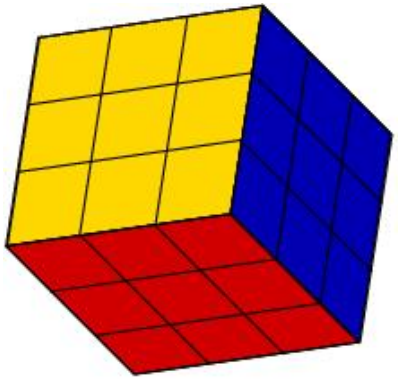
- STRIPS Action Planning (60s!)







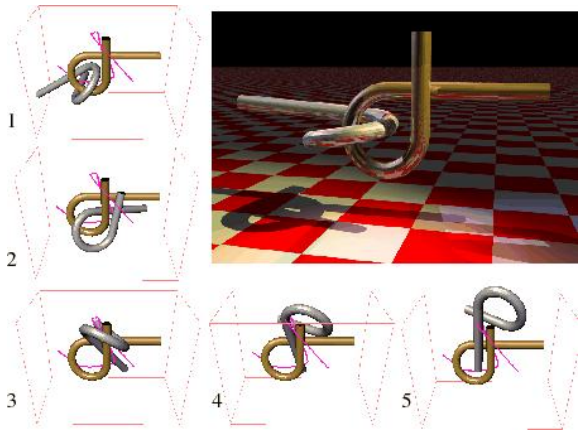
- Puzzles



(a)

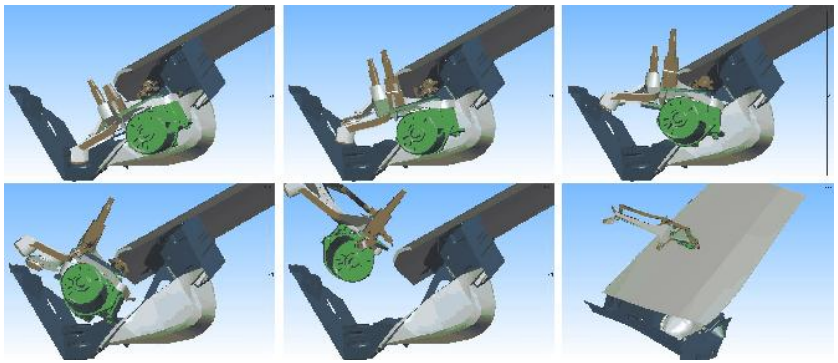
1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	

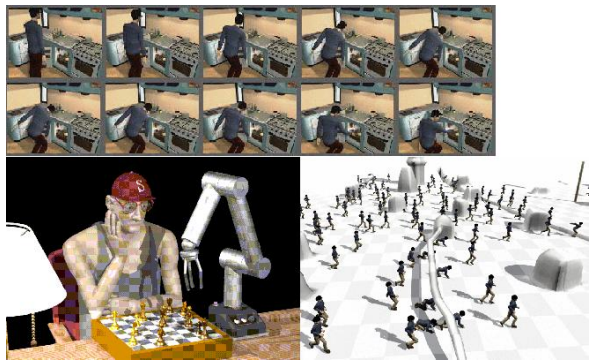
(b)



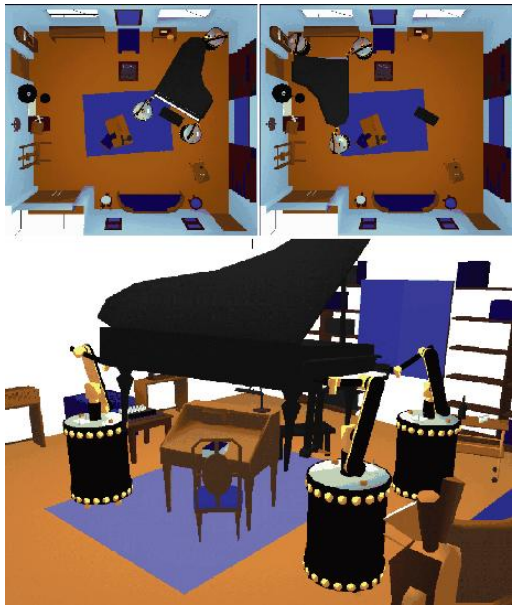
Motivation
More practical applications

- Industrial assemblies
- General motion planning





- The piano mover's problem



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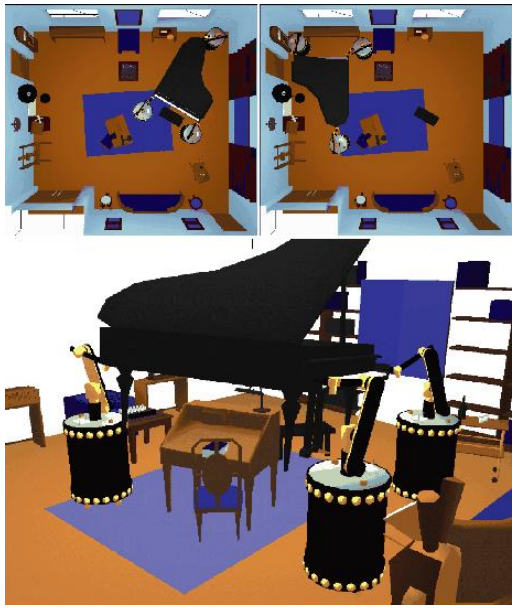

Basic Path Planning

Compute a continuous sequence of collision-free robot configurations connecting the initial and goal configurations.

Why bother?

Original 'piano mover's problem', now includes uncertainties, multiple bodies and dynamics.

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The user of the plan

Robot, decision maker, agent, software agent, controller, policy, control law, player...

Basic Ingredients

- State
  - discrete, continuous (partially known, dynamic)
- Time
  - explicitly modelled, implicit
- Actions
  - (a.k.a. inputs, controls)
- Initial and Goal state(s)
  - single or set
- Criteria
  - feasibility, optimality, ...
- Plan
  - a specific strategy or behaviour that the agent follows

Basic representation

- A world example

- Find a path between two locations in an unknown, partially known, or known environment
- Performance
  - Completeness
  - Optimality (operating cost)
  - Complexity

Strong Assumption

Some form of discretization on your state space.

Breadth first
Depth first

- Uninformed search
  - No information from environment
  - Blind search
- Informed search
  - Use evaluation function
  - More efficient (can tune)
  - Requires a heuristic

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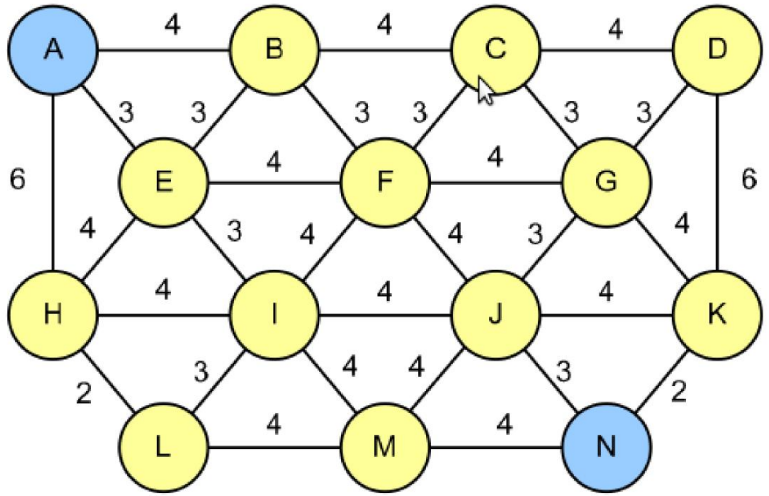
Breadth first
Depth first
Dijkstras algorithm (introduced notion of cost)
A*,D*,etc. (incorporation of heuristic estimate of the cost)
Best first
Iterative deepening
Backwards search
Incremental/Anytime versions of all of the above..
Bidirectional search

- Evaluation function: $f(n) = g(n) + h(n)$
- Operating cost: $g(n)$
  -Actual operating cost having been already traversed
- Heuristic function: $h(n)$
  - Info used to find the most promising node to traverse
  - Admissible: never overestimate the actual path cost



Heuristics

A = 14	H = 8
B = 10	I = 5
C = 8	J = 2
D = 6	K = 2
E = 8	L = 6
F = 7	M = 2
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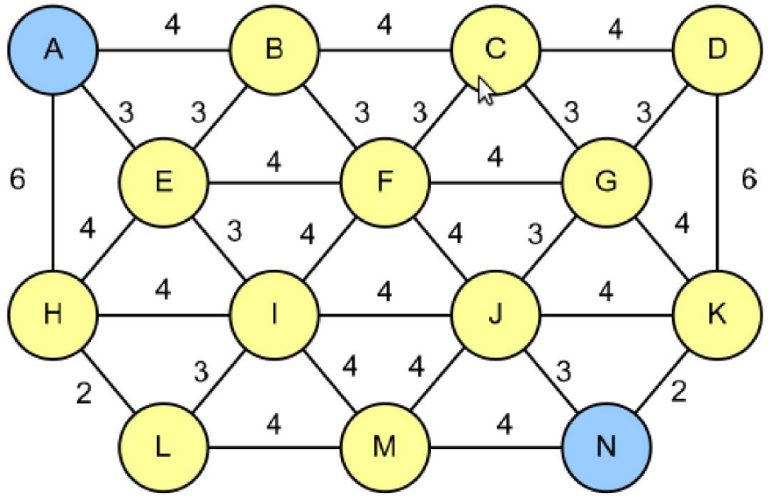

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-Effect of heuristic Dijkstra,A*,wA*.

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- A world example
- ..but what if your robot is not a point?
  - Blow up the world (i.e. take the Minkowski sum of the convex polygons)
  - Issues with rotations and translations
- 2-link robot example

Cell decompositions to get to adjacency graphs
- Two cells are adjacent if they share a common bound
- Exact and approximate
- Techniques

  - Trapezoidal Decomposition
  - Morse Cell Decomposition
    - Boustrophedon decomposition
    - Morse decomposition definition
    - Sensor-based coverage
    - Examples of Morse decomposition
  - Visibility-based Decomposition

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-Vanilla RRT example

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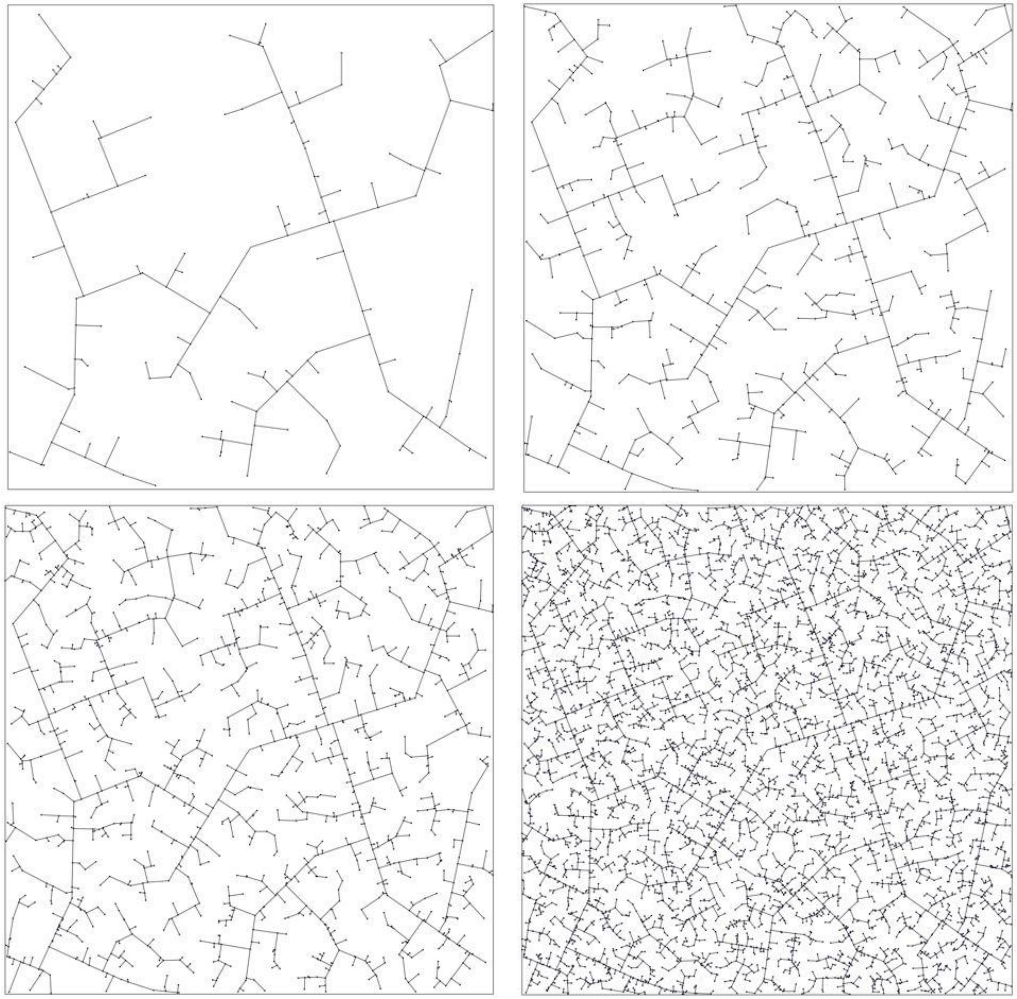
-Vanilla RRT example -Problems with RRTs

RRT example



Feedback motion planning

- Grids

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- Grids
- Vector fields

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- Barycentric interpolations

Feedback motion planning

- Grids
- Vector fields
- Barycentric interpolations
- Funnel covering

Feedback motion planning under differential constraints

Differences: 1) the introduction of time, 2) the state or phase space, and 3) the state transition equation

- Motion primitives
  e.g. manoeuvre automatons, local control laws with well defined in/out states and connectivity graph
- Reachability sets
  e.g in phase space and lattices
- Regions of inevitable collision
  e.g. it is in phase space and state dependent, depends on velocity for instance
- System simulations
  e.g. sampling control space

The beyond

System theoretic and analytical techniques

- Dynamic programming
- LQR, LQR-trees
- iLQG
- Pi^2