Geodesic trajectory generation on learnt skill manifolds

Ioannis Havoutis, Subramanian Ramamoorthy -- *School of Informatics, University of Edinburgh, UK.*

Motivation

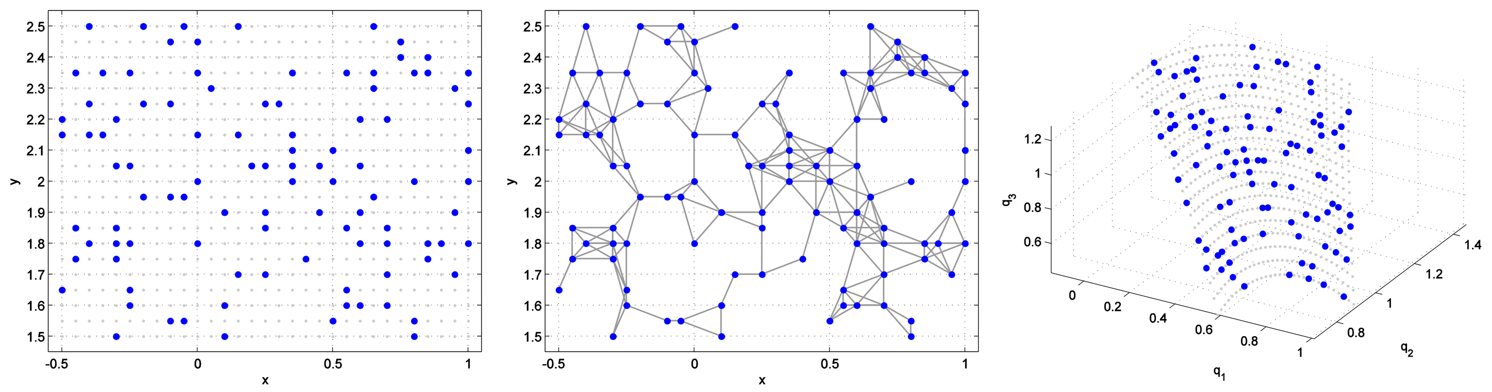
Humanoid robots are appealing due to their inherent dexterity. However, their potential benefits may only be realized with a correspondingly flexible motion synthesis procedure. Designing flexible skill representations that also capture non-trivial dynamics effects over a large domain, such as in real humanoid robots, has been an open challenge.

We present one such flexible trajectory generation algorithm that utilizes a *geometrical representation* of humanoid skills in the form of skill manifolds [1]. Such manifolds are learnt from demonstration data that may be obtained from off-line optimization algorithms or a human expert.

Manifold learning

Central to our approach is a *nonlinear manifold learning* method that is able to capture the geometrical properties of the intrinsic low-dimensional manifold, that training data points are generated from. Our learning algorithm is a modification of LSML by Dollar et al. [2].

Formally our goal is to learn a model of the tangent space of the low dimensional nonlinear manifold, conditioned on the adjacency relations of the high dimensional data.



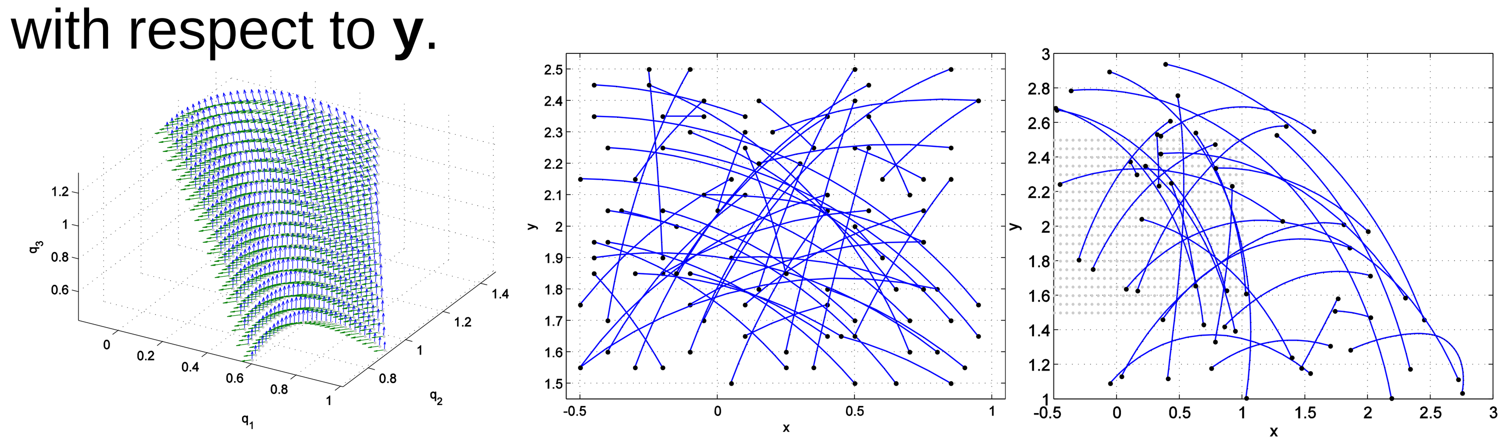
Given that our *D*-dimensional data lies on a locally smooth *d*-dimensional manifold in *D*-dimensional space, where d < D, there exists a continuous *bijective* mapping $\mathcal{M}$ that converts low dimensional points $\mathbf{y} \in R^d$ from the manifold, to points $\mathbf{x} \in R^D$ of the high dimensional space,

$$x = \mathcal{M}(y).$$

The goal is to learn a mapping from a point on the manifold to its *tangent basis* $\mathcal{H}(\mathbf{x})$,

$$\mathcal{H}:\ x \in \mathbb{R}^D \mapsto \left[\frac{\partial}{\partial y_1}\mathcal{M}(y)\cdots\frac{\partial}{\partial y_d}\mathcal{M}(y)\right] \in \mathbb{R}^{D\times d}$$

where each column of $\mathcal{H}(\mathbf{x})$ is a basis vector of the tangent space of the manifold at **y**, i.e. the partial derivative of ***M*** with respect to **y**.

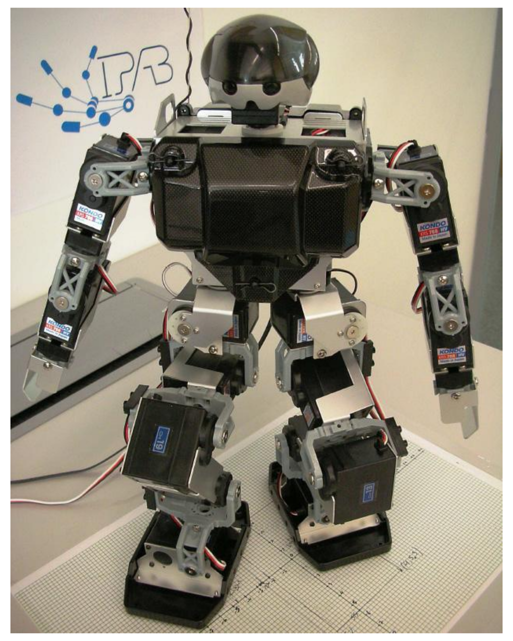


By approximating the tangent space of the manifold, we can perform geometric operations and compute geodesic paths as approximately optimal solutions.

The model allows robust generalization :

- within the region of support of the training data and
- outside the region of support of the training data, up to a reasonable distance .

Experimental Setup

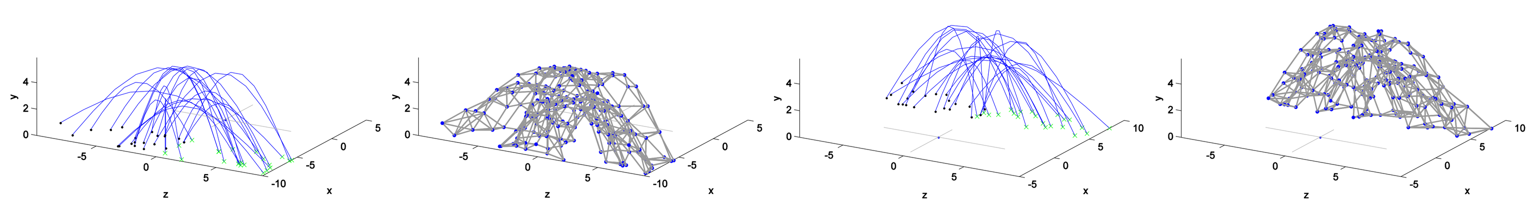


Robot: Kondo's *KHR-1HV* 19 DoF humanoid robot.

Task: walking in a reasonably large interval in *step length*, *width* and *height*.

Redundancy resolution strategy: Unconstrained nonlinear optimization problem. The cost function we designed evaluates:

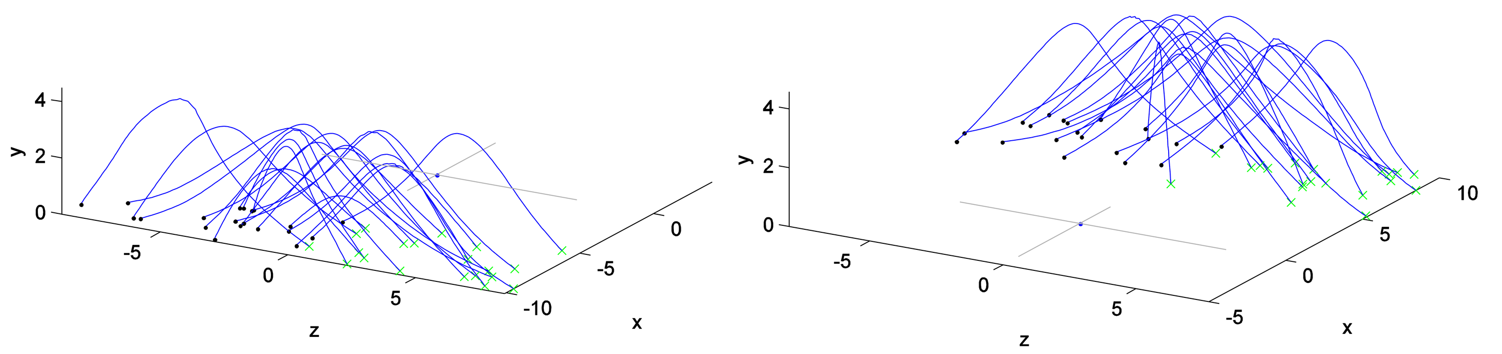
- the *distance* of the midpoint of the swing foot to the desired goal,
- the *alignment* of the swing foot with the **x** and **y** versors, to keep the foot flat,
- the horizontal *distance* of the position of the pelvis to the desired pelvic position, to manipulate the *center of mass* of the humanoid,
- the *alignment* of the waist of the robot with the **z** versor, to keep the humanoid, from the hips up, in an upright position.



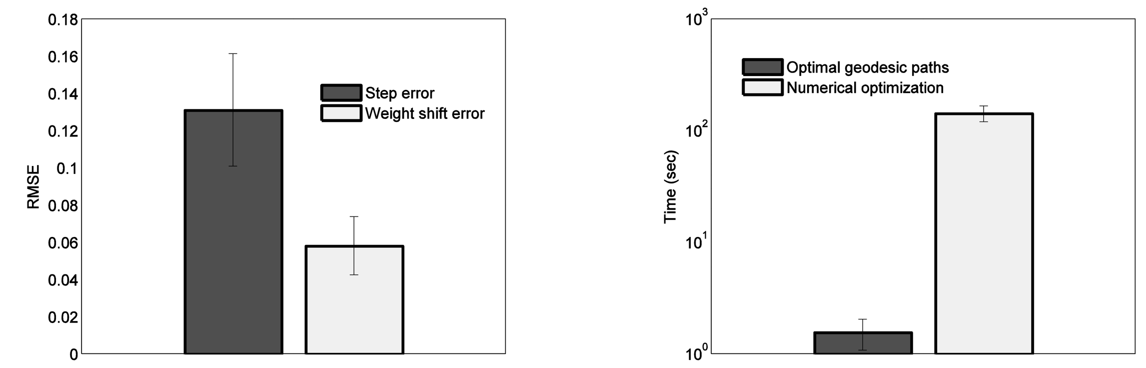
Results

The learnt manifolds are able to produce smooth walking trajectories that satisfy the optimization criteria used to produce the training data.

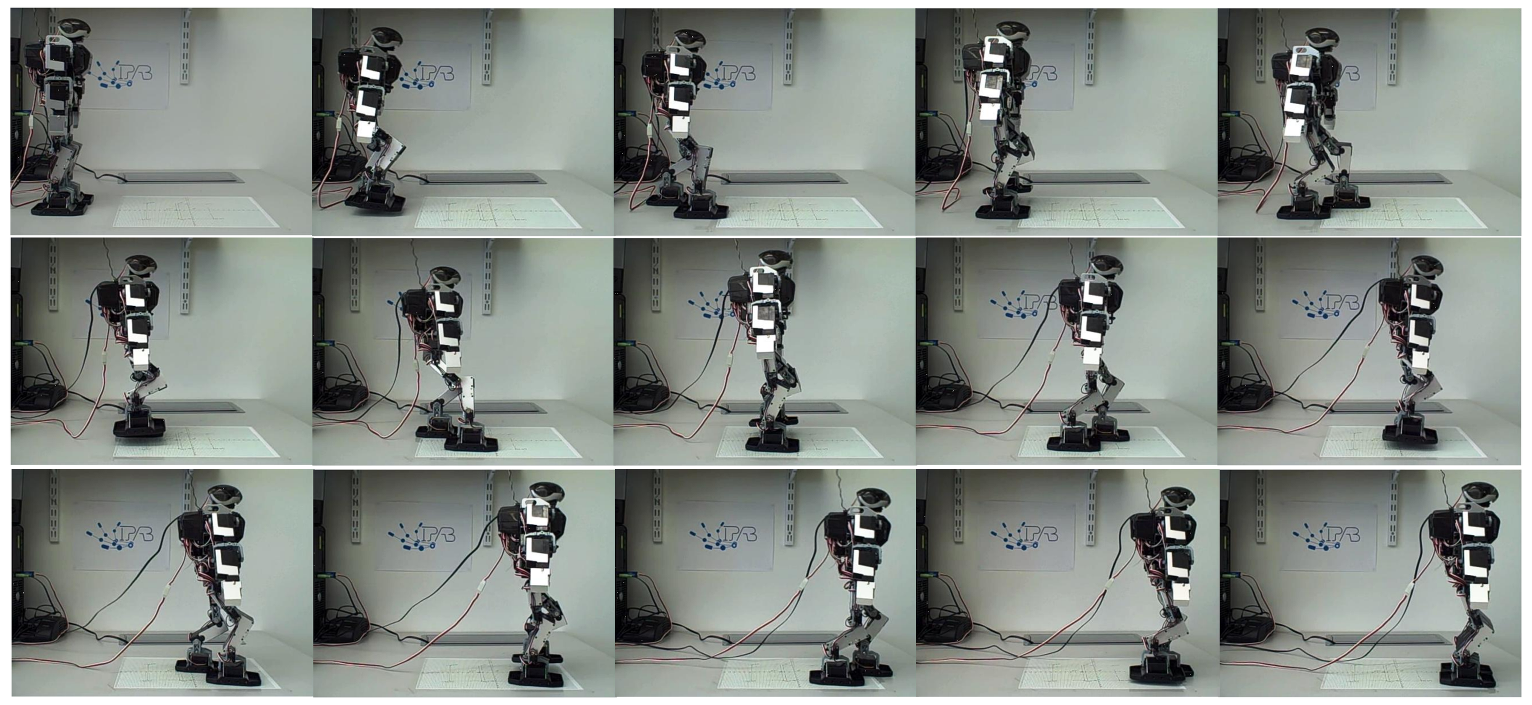
The procedure was able to produce stable walking in the *continuum* of the reachable space of the robot as depicted bellow for right and left leg swings accordingly.



We can effectively capture the geometric properties of a low-dimensional skill manifold, that underlies a high dimensional dataset, and *tightly integrate* this with the process of trajectory generation.



The approach can be *naturally* used to generate joint space trajectories that reflect the optimality and constraints inherent in the training data, thus producing novel approximately optimal solutions to *continuous path planning queries* in a fast and efficient manner.



References

[1] I. Havoutis and S. Ramamoorthy, "Motion synthesis through randomized exploration on submanifolds of configuration space," in RoboCup Int. Symp., 2009.
[2] P. Dollar, V. Rabaud, and S. Belongie, "Non-isometric manifold learning: Analysis and an algorithm," in ICML, June 2007.